Mathematical Modeling of Low Velocity Impact on Hybrid CNG Cylinder

N. Mohsin[1], R. Khan[2], S. A. Masood[3]

[1,2,3]Mechanical Engineering Department, Islamic International University, Islamabad
[1]engr_nabeel99@hotmail.com

Abstract- In this paper a comparison was studied between mathematical modeling of CNG hybrid cylinder under impact. Impact resistance of hybrid material is analyzed using analytical modeling. Impact resistance against applied force is studied using Hertzian law which relates contact stiffness of wooden block and composite. Finite Element Analysis (FEA) for Low velocity impact (LVI) of wooden block on hybrid CNG cylinder is done for finding structural response parameters. 2 degree of freedom (2DOF) spring mass system is used for mathematical modeling of impact of wooden block on hybrid CNG cylinder. Stiffness values by analytical modeling are used in mathematical modeling for comparison with FEA. Results further show that values found by FEA and mathematical modeling are closely comparable.

Keywords- Hybrid Material, FEA, CNG Cylinder, Ansys Workbench

I. INTRODUCTION

Composite material is an alternate choice for pressure vessel than metals as they are lighter, stronger and cheaper. Composite material is a preferable choice for manufacturing of the pressure vessel. The pressure vessels made from the composite materials are lighter with higher burst pressure. [i].

Use of (Compressed Natural Gas) CNG as a fossil fuel has provided advantage for the automotive industry. Hybrid cars are made with idea of fewer emissions and highly efficient. CNG cylinders is made of storage in automotive. Pressure vessels are critical to impact loading during service life. Human safety has high and strict standards for safety throughout world. High safety standards need designer to increase thickness and consequently gain weight. Fiber metal laminates can be appropriate choice with considering safety and weight constraints.

The paper is structured as follows: Section I: introduction to composite and hybrid composite CNG cylinder in automobiles. Section II: literature introduction of Fiber metal laminates (FML), its application and latest research work in impact resistance. Section III current system description, its analytical modeling, FEA simulation and mathematical modeling of impact. In Section IV: comparison is made FEA simulation and mathematical modeling results.

II . LITERATURE REVIEW:

Combining suitable properties of material and fiber reinforce composite as an idea behind for development of new type of material called fiber metal laminate (FML). Fiber metal laminates (FML) [ii] are light weights hybrid metal and composite material or multi layered heterogeneous material. It consists of alternating layers of metal bonded with composite laminates. FML demonstrate outstanding damage tolerance capabilities in combination with excellent impact resistance when compared with composite and metal separately. FML is providing impact resistance under impact. Composite has properties of heterogeneous combination of different materials. Composite has high localized densities due to fiber immersed in Epoxy and high specific strength than single material. It has also improved fatigue resistance. In FML, it is composed of alternately stacked metal with composite such that the superior fatigue and fracture characteristics associated with fiber reinforce composite material may be combined with plastic behavior and durability offered by many metal [xvi] . These laminates consist of thin (0.3mm ~ 0.5 mm) metal strip bonded together with alternating unidirectional composite prepeg. These prepeg are Aramid or glass fiber in epoxy resin as shown in Fig. 1.

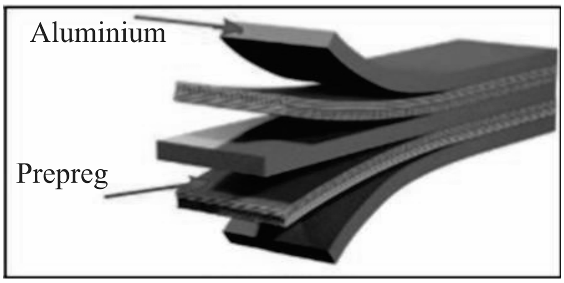

Fig. 1. FML materials layup

FML has significant characteristics of high specific strengths, better damage tolerant, blunt notch strengths, formability, and reparability.

Impact is force transferring phenomena that have to be considered in the design process for safety reasons [iii]. Impact damage is an important type of failure in modern structures. However no available method can predict the perforation behavior of FML under impact loading due to different material combination. Multiple studies have resulted in the theoretical prediction of the impact response of FML under various loading conditions but the methodologies focus mainly on the elastic response which represents a minor portion of the perforation response. The impact resistance and damage tolerance should be determined. The development of impact resistance is limited because of scatter experimental data. New technique is necessary to assess impact performance of FML.

We are concentrating attention in this paper to develop some method for theoretical study of FML. In study of impact on FML require modeling of FML . Two improve model have been developed energy based model and spring mass model for calculating impact force and duration during low velocity impact of composite plate. Both model include contact deformation of impactor ,and plate as well as bending , transverse shear, membrane deformation of plate. Energy based model is based on conservation of energy. It is need of modeling that we use conservation law for imparting impact force on contact. Conservation of energy method assumes that impactor becomes stationary when structure reaches its maximum deflection and initial kinetic energy consumes in deforming structure. This approach is beneficial that deformation energy is quantified and identified separately. It determines maximum impact force .In spring mass approach spring is used to represents effective structural stiffness of system.

This model is effective for finding impact force load time history which is required for damage tolerance design studies [ix]. 2DOF spring mass model caters membrane, flexural and transverse stiffness. It contains mass of impactor and plate. In this spring mass model composite orthotropic nature of plate is considered. Spring mass model consider contact of impactor with plate and consider contact force duration for plate deflection [ix].

III. SYSTEM UNDER STUDY

GEOMETRY OF CNG CYLINDER:

Hybrid CNG cylinder is used for the storage of CNG at high pressures. Hybrid CNG cylinder consists of metal liner wrapped in composite cylinder. In this paper, CNG cylinder is made of 30CrMnSiA annealed Metallic liners with the thickness of 0.5 mm. It is wrapped by Epoxy glass fiber composite having thickness of 4.4 mm for making it light weight with high strength as shown in Fig. 2.CNG cylinder simulation is done according to safety standards of FMV404 i.e. American automobile safety. CNG cylinder is pressurized to 360 bars. Impactor is wooden block having mass 18 kg and velocity of 12 m/sec as shown in Fig. 2.

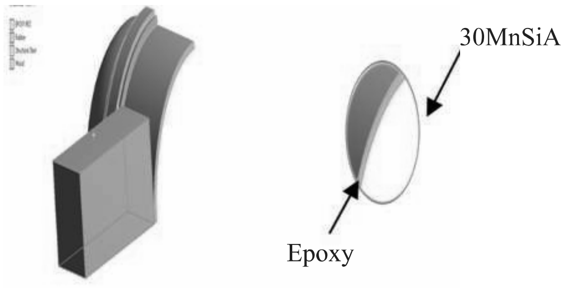

Fig. 2. Hybrid Cylinder and Impact Object

Many models used to study impact dynamics are classified according to impact velocity of projectile. There is not any generic model for describing impact of composite as well as hybrid structure. In order to understand mechanics of composite and hybrid structures it is desirous to study impact response of composite and hybrid structures [iv].

This paper is concerned about modeling of impact on CNG hybrid cylinder. Analytical modeling of composite and metal determine stiffness and deformation values of composite.

Impact phenomena broadly can be divided into three categories based on velocity at time of impact. Low velocity impact (LVI) range from (0~12m/sec). Global plate motion is established in LVI [v]. LVI induce mild damage in composite laminates. This invisible damage caused by mild impact was found to decrease residual strengths. Intermediate velocity impact regimes ranges from (12 ~50 m/sec). Hurricane debris collision on runway or roads lies in this category. High velocity impact (Ballistic impact) is usually results of explosive blasts, bullet strike[vi-viii] .

Considering mechanics of impact produce shear wave, flexural wave and tension or compression waves are produce in plate [ix]. In LVI impact stress wave propagation is slow and shear wave rebounds after reaching boundary condition [x]. Shear wave changes into flexural waves as more time of impact passes. Tensile or compressive strength at point of impact becomes dominant for composite [xi].

In our study case composite material in CNG hybrid cylinder bears impact directly. After damage in composite layer impact is transferred on metal liner. Metal liner deformation is produced after composite laminate damage. In this paper using analytical equations we modeled composite and metal separately. All symbols used in analytical modeling are described in Table 1 in Appendix A at end of paper.

Analytical Modeling

Contact within composite laminates and metal is essentially important parameter for transferring of impact force on both materials. During LVI composite deforms more easily so contact area between impactor and composite changes non -linearly. Transient deformation of the panel as shear wave propagate from the point of impact is approximated by spring mass system. Local deformation in the contact zone is not modeled with beam, plate or shell theories usually assume that the structure is inextensible in the transverse direction. However in many cases local indentation has a significant effect on the contact free history and must be accounted for in the analysis. During loading phase contact force P is related to indentation α by Hertzian law [xii] expressed in equation 1,2,3.

$$P = k\alpha^{1.5} \quad (1)$$

$$P = \left(\frac{5}{4}\right)^{\frac{3}{5}} [M^3 V^6 k^2]^{1/5} \quad (2)$$

$$T_c = 3.1245[\frac{M^2}{Vk^2}]^{1/5} \quad (3)$$

Where $\mathbf{T_c}$= contact time (sec)

Basically stiffness in glass epoxy composite is calculated using material properties of Glass epoxy given in Table II.

TABLE II
ORTHOTROPIC PROPERTIES OF GLASS/EPOXY

Material properties	Glass Epoxy
E_1	38.6 Gpa
$E_{zz} = E_{rr}$	8.27 Gpa
$G_{\Theta z} = G_{\Theta r}$	4.14 Gpa
G_{rz}	4.14 Gpa
$V_{\theta r} = V_{\theta z} = V_{rz}$	0.26
ρ	1800 kg/m^3

A11, A22 and A12 are extensional stiffness parameter in longitudal, transverse and shear direction respectively for glass epoxy composite. The contact stiffness is given by equation 4, 5, 6, 7, 8, 9 & 10.

$$K_1 = 1 - \frac{\vartheta_1^2}{\pi E_1} \quad (4)$$

$$K_2 = \frac{\sqrt{A_{22}}[(A_{11}*A_{22}+G_{zr})^2-(A_{12}+G_{zr})^2]^{1/2}}{2\pi\sqrt{G_{zr}}(A_{11}A_{22}-A_{12}^2)} \quad (5)$$

$$A_{11} = E_z(1-\vartheta_r)\beta \quad (6)$$

$$A_{22} = \frac{E_r\beta(1-\vartheta_{zr\delta}^2}{(1+\vartheta_r)} \quad (7)$$

$$A_{12} = E_r\vartheta_{zr}\beta \quad (8)$$

$$B = \frac{1}{1-\vartheta_r-2\vartheta_{zr\delta}^2} \quad (9)$$

$$\delta = \frac{E_r}{E_z} \quad (10)$$

$$\lambda = k_c^{2/5} V^{1/5} M^{3/5} / [8\sqrt{m^-} D^*] \quad (11)$$

Using equation 1, 2 , 3 & 11 we get following parameters in Table III.

TABLE III
CONTACT FORCE PARAMETERS

Contact parameters	Glass Epoxy
P	1665 N
T_c	0.148 sec
λ	614.2e6

λ Inelasticity parameter comes out by putting values in equation (11) which shows that most of portion of kinetic energy is absorbed by glass epoxy composite.

Contact area of composite increases with penetration as shear wave interacts with bending waves. This material behavior can be approximated with two degree of freedom spring mass system.

Energy Based Model

Energy based modeling is based upon the principle of conservation of total energy of plate impactor system [xiii]. Total energy is sum of energies due to contact, bending, shear and membrane deformations and plastic energies petaling of aluminum layers. All energy which is equivalent to impactor kinetic energy [xiv] as shown in equation 12.

$$E_{impactor} = E_c + E_{bs} + E_m + E_p \quad (12)$$

$\mathbf{E_c}$= Contact Energy
$\mathbf{E_{bs}}$= Energy due To bending and shear
$\mathbf{E_m}$= Membrane Energy
$\mathbf{E_p}$= Plastic Deformation Energy

Contact energy is based upon on integral of contact force producing indentation α using equation 13.

$$E_C = \frac{\frac{2}{5}P^{5/3}}{n^{2/3}} \quad (13)$$

Reactive force can be resolved into two components using equation (14)

$$P = P_{bs} + P_m \quad (14)$$

$\mathbf{P_{bs}}$ is reactive force which is due to bending and shear deformation. $\mathbf{P_m}$ is reactive force associated with membrane deformation. Equation of 15 can be further elaborated as

$$P = K_{bs}w + K_m w^3 \quad (15)$$

$$K_{bs} = \frac{K_s K_b}{K_b + K_s} \quad (16)$$

$$K_b = \frac{4\pi E_r h^3}{3a^2 * 1 - \vartheta_r^2} \quad (17)$$

$$K_s = 1.33\pi G_{zr} h\left(\frac{E_r}{E_\theta} - 4\vartheta_{rz}G_{zr}\right) * \frac{1}{1.33 + \frac{\log a}{a_c}} \quad (18)$$

$$E_{bs} = \frac{1}{2K_{bs}w^2} \quad (19)$$

$$E_m = \frac{1}{4K_m w^4} \quad (20)$$

$$E_p = \frac{n_p(\sigma_o \pi^2 R h_{steel}{}^2)}{4} \quad (21)$$

where K_b, K_s ,K_m are bending ,shear, membrane stiffness of composite respectively. Energy absorption during impact can be elaborated in Table IV.

TABLE IV
ABSORBED ENERGY PARTITION

Energy	Glass Epoxy	% energy absorbed
E_m	4.67e-5 J	3.60e-6 %
E_c	753.19 J	58.1 %
E_{bs}	3.024 J	0.23 %
E_P	544.768 J	42.0
E_{tot}	1296 J	100 %

Using energy balance equation

$$\mathbf{M_1 V_o^2 = K_{bs} w^2 + \lceil(\frac{K_m w^4}{2} + \frac{\frac{4}{5}\lfloor(K_{bs}w + K_m w^3)\rfloor}{n2}\rceil^{1/3}} \quad (22)$$

By putting values K_b, K_s,K_m in above equation and using Newton Raphson numerical technique the deflection w is calculated, w is **25.9** mm by finding roots of above equation. The deflection is crossing over material thickness. This value is indication of deflection which is bearing by composite only. This deflection is reduced by using metal liner along with composite absorbing 42 % of total energy.

Finite Element Simulation

CNG cylinder in simulation is carried by following American standards of automobile safety FMV404. CNG cylinder is pressurized to 360 bars when filled. Impactor is wooden block having mass 18 kg and velocity of 12 m/sec. In FEA simulation ANSYS EXPLICT is used for impact simulation. For impact simulation we used proximity based body algorithm in ANSYS Explicit dynamics module. From impact simulation basically target is to calculate impact penetration in composite and metal liner. Material properties of Wood is used in ANSYS is given in Table V.

TABLE V
MATERIAL PROPERTIES OF WOOD

Material properties	Glass Epoxy
E	0.9 Gpa
E	8.27 Gpa
ν	0.45
ρ	666 kg/m^3

In ANSYS Explicit dynamics hybrid CNG cylinder is modeled having impact by wooden block. Hybrid layup has 4.4 mm thick composite layup made in ANSYS ACP and 0.5 mm metallic liner. CNG cylinder is pressurized on 360 bars. For holding cylinder metallic strips and rubber bands are used for simulation. It has been observed that deflection stress in composite metal laminate. It has shown in Fig. 3.

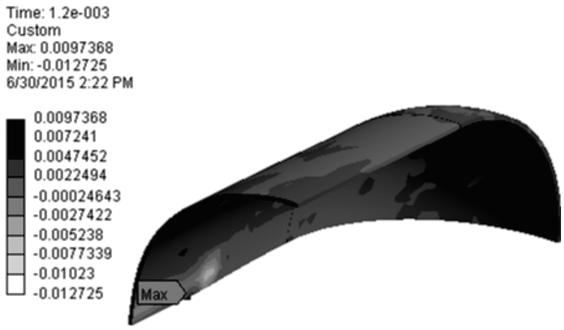

Fig. 3: Strains in the pressure vessel

Mathematical Modeling in MATLAB

Applying Newton second law of motion, equation of equilibrium of two degree of freedom spring mass system are written as in equation 22 & 23 [xv]

$$M_1\ddot{x} + \lambda n[(x_1 - x_2)]^{1.5} = 0 \quad (22)$$

$$M_p\ddot{x} + K_{bs}x_2 + K_m x_2^3 - \lambda n[(x_1 - x_2)]^{1.5} = 0 \quad (23)$$

Mathematical modeling of composite laminate using spring mass system which is governed by equation 24

$$F = M\ddot{x} + C\dot{x} + Kx \quad (24)$$

When contact behavior follows Meyer's law and overall deflection are negligible a (single degree of freedom) SDOF model including damping is given in equation 25.

$$F = M\ddot{x} + C\dot{x} + Kx^n \quad (25)$$

Spring mass system has two degree of freedom. It is dealing with rigid body of impactor and plate. This model is extension of Lee's spring mass model for impact of beams. The spring combination below the plate mass satisfy the following conditions impact force is shared by bending shear and membrane deformations of plate; for thin plate the spring combination reduced to thin plate theory due to relatively low bending stiffness.

Let $X_1(t)$ and $X_2(t)$ as shown in Fig. 4 represents the displacement responses of the two masses at any time t after impact. The corresponding velocities were represented by differential of displacement of each mass individually. The contact deformation is given by

$$\dot{x}_1(t)-\dot{x}_2(t) \qquad (26)$$

Throughout the analysis the impactor mass was assumed to be in contact with the plate. Initial conditions for equation are X_1 (0) = 0 and X_1 = V_o for impactor mass and X_2 (0) = 0 and X_2 (0) = 0 for plate mass. The coupled numerical equation is solved in MATLAB. Simulink model is shown in Fig. 5.

In graph of MATLAB simulation in Glass epoxy is 1.27mm as shown in Fig. 6. In this simulation metallic plate is not consider.

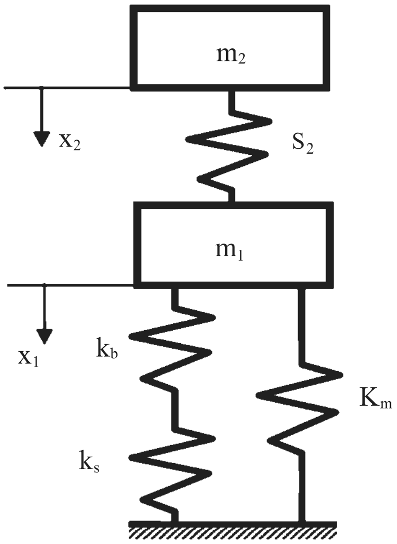

Fig. 4. Graphical description of 2 DOF spring mass model

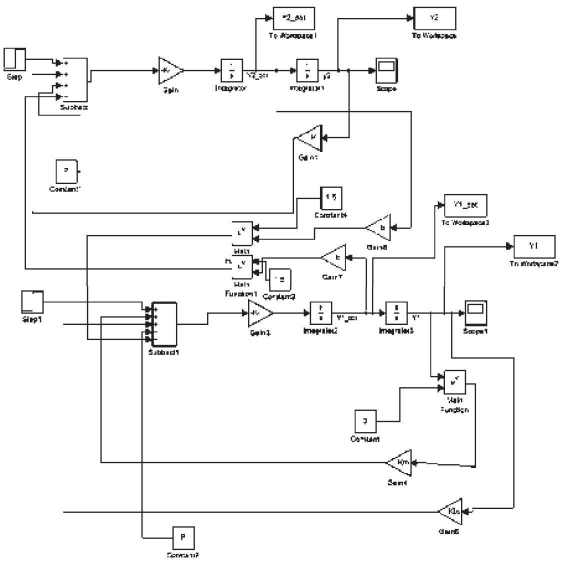

Fig. 5. 2DOF Spring Mass Model in MATLAB SIMULINK

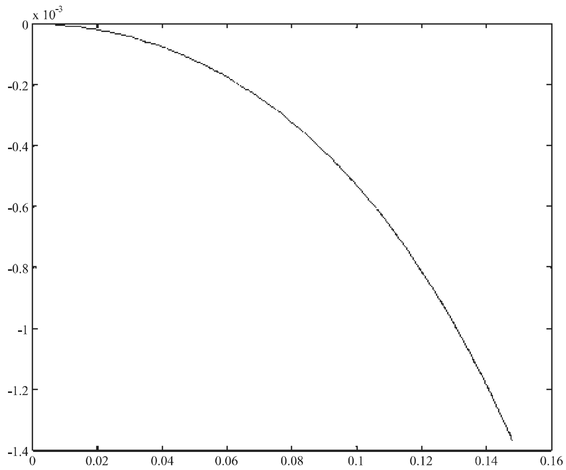

Fig. 6. Displacement time history in Glass/Epoxy plate after impact of wooden block by using spring mass model

IV. CONCLUSION

Comparing all methods in this paper we consider discretization of differential equation of impact on composite cylinder. This equation has given deformation of composite cylinder i.e. 1.27 mm. In FEA this simulation deformation in hybrid structure deformation is 0.44 mm. The difference is due to pressurized CNG cylinder. In paper energy based simulation reveals differentiation of energy distribution in different mechanisms in impact. Metallic liner takes 42 % impact energy which is less than plastic deformation energy .i.e. 1665.49 J from equation 21. From analytical equation deformation comes is 25.4 mm because considering only composite in equation 22.

Concluding all methods give impact dynamics depends upon different methods mentioned in analytical, energy, mathematical modeling method. In FEA simulation it is able to get actual boundary conditions. Results haves shown close relation to each method.

REFERENCES

[i] Abrate Serge. "Modeling of impacts on composite structures." *Elesvier-Composite structures*, 1998: PP 130-138.

[ii] CAJR, Vermeeren. "A historic overview if the developement fiber metal laminates ." *applied Composite materials*, 2003 vol 10 No. 4: pp 189-205.

[iii] E. H. LEE. "Impact of mass striking a beam." *Applied mechanics* . Dec 1940. 1-129 to A138.

[iv] F. D. Moriniere R. C. Alderliesten, R. Benedictus. "energy distribution in Glare and 2024-T3 aluminum during low velocity

impact." *ICAS 2012.* Delft , The Netherlands: delft techonolgies, 2012.

[v] G. Caprino, Spataro S. del Luongo. "Low velocity impact behavior of fiber glass aluminum laminates ." *compos part A*, 2004: 605-615.

[vi] Hoo Fatt MS, Park KS,. "Modeling low velocity impact damage of composite sandwich panels." *J sandwich struct Material* , 1985,23: : 442-9.

[vii] Iaccarino P, Langella A and Caprino G. "A simplified model to predict the tensile and shear stress strain behavior of fiber glass aluminum laminates." *Composite science and technology* , 2007: 1784-1793.

[viii] J. W., Vlot A. and Gunnink. *Fiber metal laminates an introduction.* Klyuwer Academic publisher, 2001.

[ix] K. N. Shivaku, ar, W.Elber and W.Illg. *Prediction of Impact force and duration during low velocity impact on circular composite laminates.* NASA Technincal memorandum, Hampton Virginia 23665: NASA Langley research center, October 1983.

[x] Logan, D.J. *Applied Mathematics* . Jhon Wiley & Sons, second edition 1997.

[xi] "Mettalic materials and elements for aerosapce vehicle structures." In *MIL HDBK-5H*. 1998.

[xii] Payeganeh GH, Asshenai Ghasemi F and Malekzadeh. "Dynamics response of fiber metal laminates subject ti low velocity impact." *thin walled structure*, 2010: pp 62-70.

[xiii] Vadori, G.Belingardi R. "Low velocity impact tests of laminates glass fiber epoxy matrix composite material plates." *Interantion journal impact Eng.*, 2002: 213-229.

[xiv] Vlot A, Kroon E and La Rocca G. "Impact response of fiber metal laminates." *Key engineering material* , 1998: pp 235-276 .

[xv] W. J. Strong. *Impact Mechanics.* Cambridge university Press UK, 2000.

[xvi] Remmers JC .Discontinuities in material and structures: a unified computational approach , PhD thesis , Delft university of technology , 2008

[xvii] M. Sadighi. impact resistance of fiber metal laminates a review , international journal of impact engineering 49 (2012) 77-90

[xviii] C.C FOO, A modified energy balance approach model to predict low velocity impact response for sandwich composite, composite structures 93 (2011) 1385 - 1393

Appendix A

TABLE I
LIST OF SYMBOLS

α =Indentation in Composite Panel	K_m =Membrane Stiffness
A_{deb} =De-bonding Area	L =Length of Indenter
A_{del} =Delamination Area	R_o =Equivalent Projectile Radius
A_{ij} =Membrane Stiffness	t =Time
D_{ij} =Bending Stiffness	U =Strain Energy
E =Young's Modulus	V_o =Projectile Initial Velocity
Ep =Plastic Modulus	w =Transverse Plate Deflection
E_{ay} =Average Panel Stiffness	W =Work done by External Force
E_{bm} =Bending and Membrane Energy	z =Through Thickness Co-ordinate
E_{deb} =De-bonding Energy	ϵ =Strain
E_{del} =Delamination Energy	ϵ_{cr} =Fracture Strain
E_{ij} =Laminate Stiffness	ϵ_{Al} =Fracture Strain in Aluminum
E_p =Petaling Energy	$\epsilon_{G/E}$ =Fracture Strain in Glass Epoxy
e_t =Energy density for tensile failure	ν_{ij} =Poisson's Ratio for Laminate
E_t =Tensile fracture energy	ν =Poisson's Ratio for Panel
E_{tot} =Total Energy	Π =Total Potential Energy
F =Impact Load	ρ_{av} =Average Density
G_{ij} =Laminate Stiffness	ρ =Density
h =Laminate Thickness	σ_f =Tensile Fracture Strength
h_{AL} =Aluminum Ply Thickness	σ_o =Yield Strength
$h_{G/F}$ =Glass Epoxy Ply Thickness	ξ =Deformation Zone
K_b =Bending Stiffness	